Dynamic Sets and Searching

- Analysis Technique
  - **Amortized Analysis**
    // average cost of each operation
    in the worst case
- Dynamic Sets
  // Sets whose membership varies during computation
  - **Array Doubling**
  - **Implementing Stack with array doubling**
- Searching
  // Exist or not, in where
  - **Binary Search Trees**
  - **Hashing**

TECH
Computer Science

Amortized Analysis

- Provides *average* cost of each operation
  in the worst case for successive operations
- Aggregate method
  - **show for a sequence of n operations takes worst-case time T(n) in total**
  - **In the worst case, the average cost, or amortized cost, per operation is therefore T(n)/n**
- Accounting method // spreading a large cost over time
  - **amortized cost = actual cost + accounting cost**
  - **assign different accounting cost to different operations**
    - 1. the sum of accounting costs is nonnegative for any legal sequence of operations
    - 2. to make sure it is feasible to analyze the amortized cost of each operation

Array Doubling

- We don’t know how big an array we might need when the computation begins
- If not more room for inserting new elements,
  - **allocating a new array that is twice as large as the current array**
  - **transferring all the elements into the new array**
- Let t be the cost of transferring one element
  - **suppose inserting the (n+1) element triggers an array-doubling**
  - **cost t*n for this array-doubling operation**
  - **cost t*n/2 + t*n/4 + t*n/8 + … for previous array-doubling, i.e. cost less than t*n**
  - **total cost less than 2t*n**
  - **The average cost for each insert operation = 2t**

Implementing Stack with array doubling

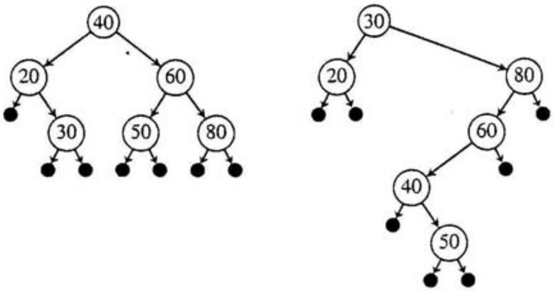
- Array doubling is used behind the scenes to enlarge the array as necessary
  - **Assuming actual cost of push or pop is 1**
    - when no enlarging of the array occurs
  - **the actual cost of push is 1 + t*n**
    - when array doubling is required
- Accounting scheme, assigning
  - **accounting cost for a push to be 2t**
    - when no enlarging of array occurs
  - **accounting cost for push to be –t*n + 2t**
    - when array doubling is required
- The amortized cost of each push operation is 1+2t
- From the time the stack is created, the sum of the accounting cost must never be negative.

Searching: Binary Search Trees

- Binary Search Tree property
  - **A binary tree in which the nodes have keys from an ordered set has the binary search tree property**
  - **if the key at each node is greater than all the keys in its left subtree and**
  - **less than or equal to all keys in its right subtree**
  - **In this case the binary tree is called a binary search tree**
- An inorder traversal of a binary search tree produces a sorted list of keys.

Binary Search Trees, e.g.

- Binary Search trees with different degrees of balances
- Black dots denote empty trees



Binary Search Tree Retrieval

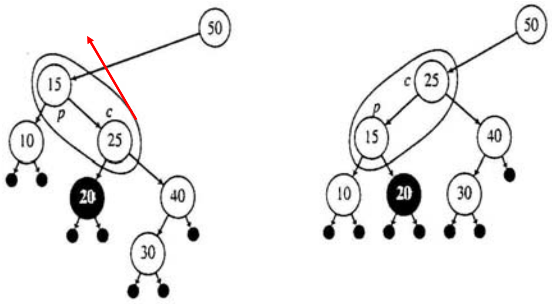
- Element bstSearch(BinTree bst, Key K)
  - **Element found**
  - **if (bst == nil)**
    - found = null;
  - **else**
    - Element root = root(bst);
    - If (K == root.key)
      - found = root;
    - else if (K < root.key)
      - found = bstSearch (leftSubtree(bst), K);
    - else
      - found = bstSearch(rightSubtree(bst), K);
  - **return found;**

Analysis of Binary Search Tree Retrieval

- use the number of internal nodes of the tree that are examined which searching for key
  - **let it be n**
- For a long chain tree structure, θ(n)
- For a tree as balanced as possible, θ(lg n)
- >> The objective is to make the tree as balanced as possible
  - **Technique: Binary Tree Rotations**

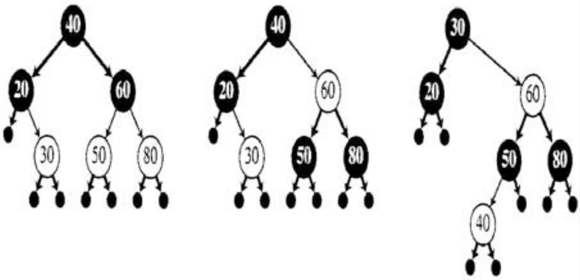
Binary Tree Rotations

- Left Rotation on (15, 25)



Making binary search trees as balanced as possible

- Red-Black Tree
  - **Let T be a red-black tree with n internal nodes. Then no node has depth greater than 2 lg(n + 1).**



Hashing to aid searching

- Imagine that we could assign a unique array index to every possible key that could occur in an application.
  - **locating, inserting, deleting elements could be done very easily and quickly**
  - **key space is much too large**
- The purpose of hashing is to translate (by using *hash function*) an extremely large key space into a reasonable small range of integers (called *hash code*).
- Hash Table
  - **an array H on indexes (*hash code*) 0, …, h-1**
  - **hash function *maps* a key *into* an integer in the range 0, …, h-1**
  - **Each entry may contain one or more keys!**
    - Hash function is a many-to-one function

Hash Table, e.g.

- data k: 1055, 1492, 1776, 1812, 1918, and 1945
- hash function
  - **hashCode(k) = 5k mod 8**
- hashCode: key
  - **0: 1776**
  - **1:**
  - **2:**
  - **3: 1055**
  - **4: 1492, 1812 // Collision!**
  - **5: 1945**
  - **6: 1918**
  - **7:**

Handling Collisions: Closed Address Hashing

- H[i] is a linked list
- hashCode: key
  - **0: -> 1776**
  - **1: ->**
  - **2: ->**
  - **3: ->1055**
  - **4: ->1492 -> 1812**
  - **5: ->1945**
  - **6: ->1918**
  - **7: ->**
- To search a given key K, first compute its hash code, say i, then search through the linked list at H[i], comparing K with the keys of the elements in the list.

Analysis of Closed Address Hashing

- Searching for a key
- Basic Operation: comparisons
  - **Assume computing a hash code equals *a* units of comparisons**
  - **there are total n elements stored in the table,**
  - **each elements is equally likely to be search**
- Average number of comparison for an unsuccessful search (after hashing) equal
  - **$A_u(n) = n/h$**
- Average cost of a successful search
  - **when key i = 1, …, n, was inserted at the end of a linked list, each linked list had average length given by (i – 1)/h**
  - **expected number of key comparisons = 1 + comparisons make for inserting an element at the end of a linked list**
  - **$A_s(n) = 1/n \sum\{i=1 \text{ to } n\}(1 + (i - 1)/h) = 1 + n/(2h) + 1/(2h)$**

Assuming uniformly distribution of hash code

- hash code for each key in our set is equally likely to be any integer in the range 0, …, h-1
- If n/h is a constants then
  - **O(1) key comparisons can be achieved, on average, for successful search and unsuccessful search.**
- Uniformly distribution of hash code depends on the choice of Hash Function

Choosing a Hash Function

- // for achieve uniformly distribution of hash code
- If the key type is integer
  - **hashCode(K) = (a K) mod h**
- Choose h as a power of 2, and h >= 8
- Choose a = 8 Floor[h/23] + 5
- If the key type is string of characters, treat them as sequence of integers, k1, k2, k3, …, k*l*
  - **hashCode(K) = $(a^{l}\,k1 + a^{l-1}\,k2 + \ldots + a\,kl)$ mod h**
- Use array doubling whenever n/h (called load factor, where n is the number of elements in the table) gets high, say 0.5

Handling Collisions: Open Address Hashing

- is a strategy for storing all elements in the array of the hash table, rather than using linked lists to accommodate collisions
  - **if the hash cell corresponding to the hash code is occupied by a different elements,**
  - **then a sequence of alternative locations for the current element is defined (by *rehashing*)**
- Rehashing by linear probing
  - **rehash(j) = (j+1) mod h**
  - **where j is the location most recently probed,**
  - **initially j = i, the hash code for K**
- Rehashing by double hashing
  - **rehash(j, d) = (j + d) mod h**
  - **e.g. d = hashIncr(K) = (2K + 1) mod h**
  - **// computing an odd increment ensures that whole hash table is accessed in the search (provided h is a power of 2)**

Open Address Hashing, e.g. Linear probing

- hashCode: key
  - **0: 1776**
  - **1:**
  - **2:**
  - **3: 1055**
  - **4: 1492**
  - **5: 1945**
  - **6: 1918**
  - **7:**
- Now insert 1812, hashcode(1812) = 4, i.e. i = 4
  - **h = 8, initially j = i = 4**
  - **rehash(j) = (j+1) mod h**
  - **rehash(4) = (4+1) mod 8 = 5 // collision again**
  - **rehash(5) = (5+1) mod 8 = 6 // collision again**
  - **… put in 7**

Retrieval and Deletion under open addressing hashing

- Retrieval procedure imitates the insertion procedure, stop search as soon as emptyCell is encountered.
- Deletion of a key
  - **cannot simply delete the the key and assign the cell to emptyCell // cause problem for retrieval procedure**
  - **need to assign the cell to a value indicating "obsolete"**